TIME-OPTIMAL MOTION PLANNING FOR ROBOTS

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ABSTRACT

At the practical application of robots, the part processing time has a key role. The part processing time is an idea borrowed from manufacturing technology. In robotics it mostly means that the robots tool-centre point should go along some given path and the tool orientation in every point should also have the given values. These two requirements have to be satisfied at the same time. In the present paper we propose a method which provides the motion in every point of the path with the possible maximum velocity. In fact, we divide the path to transient and cruising parts and require the maximum velocities only for this second part. The given motion is called "Time-optimal cruising motion". We demonstrate on an example the simplicity of the approach. Using the parametric method of motion planning, we give the equations for determining time-optimal motions. Not only the translation motions of tool-centre points but, also the orientation motions of tools may be optimally planned. The time-optimal motion planning is also possible for free paths (PTP motions). A general approach for this problem is proposed, too. In the paper, some parts are devoted to the deeper understanding of the optimization problems.

Keywords: Robot motion planning, Path planning, Trajectory planning, Parametric method, Path length, Time-optimal, Cruising motions, Tool-centre point, Orientation changes, PTP motions, Free paths

1. INTRODUCTION

Robot motions may be described by the Lagrange's equation

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\dot{\mathbf{q}},\mathbf{q}) = \boldsymbol{\tau} \qquad (1.1)$$

Where $\mathbf{H}(\mathbf{q})$ is the inertia matrix of the robot, $\mathbf{q}$ is the vector of joint displacements, their components form the joint coordinates, $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ are the joints velocity and acceleration vectors. The function $\mathbf{h}(\dot{\mathbf{q}},\mathbf{q},t)$ is the nonlinear term containing centrifugal, Coriolis, gravitational forces, frictions and also the external forces affecting the robot joints (including the forces (moments) acting at the end-effectors, too), τ is the vector of joint torques. The components of τ torques (forces) are restricted by the torques characteristics of the driving motors. The $\dot{\mathbf{q}} = (\dot{q}_1, \dot{q}_2 ....\dot{q}_n)^T$ components of the joint velocity vector are constrained by the possible maximum number of rotations (in time unit) of the motors. As it is well known, the maximums of torques (forces) are the decisive factors to determine optimal (dynamical) processes. As it will be clear from what is detailed below the constraints of joint velocities determine the time-optimal cruising motions.
Formulating an optimization problem (for example: to move the robot end-effector centre-point from one point to another in the space in minimum time), it can be solved by using the mathematical theory of optimal processes: the Pontriagin's maximum principle, or Dynamic programming of R. Bellman, or other methods.
Let us return to the Lagrange's equation. In extended form it is

$$u_i = \sum_{j=1}^{n} I_{ij} \ddot{q}_j + \sum_{j=1}^{n}\sum_{k=1}^{n} C_{ijk} \dot{q}_j \dot{q}_k + \sum_{j=1}^{n} R_{ij} \dot{q}_j + g_i \quad (1.2)$$

i=1.2,...,n

$$u_{i\,\min} \le u_i \le u_{i\,\max}$$

In (1.2):

I_{ij} are the components of the inertia matrix, C_{ijk} are coefficients for the Coriolis and Centrifugal forces. These terms are (usually) also nonlinear functions of joint displacements.

R_{ij} is the viscous damping coefficient, and g_i is the gravitational term,

u_i is the force or torque given by the actuator of the i-th joint.

In (1.2) the external forces are not indicated (but, when it is needed, they can be included). It is not indicated either that the components of joint velocities vectors are constrained, too.

Solving the optimal control problem, one may have the solution in form

$$\mathbf{u} = \tau = \mathbf{u}_{\mathbf{opt}}(\dot{\mathbf{q}}, \mathbf{q}, t) \quad (1.3)$$

It can be realized in computed torque manner. But in control practice it is desirable to solve the synthesis problem and generate the control signals depending on the error signals.

The error signals are:

$$\varepsilon_i = q_{id} - q_i \qquad i=1,2,\ldots\ldots n \qquad (1.4)$$

The q_{id} signals are the desired values (functions) of the joint coordinates.

Their derivatives are: $\dot{\varepsilon}_i, \ddot{\varepsilon}_i$ etc.

Looking at Equation (1.2) (having in mind that the coefficients are also highly nonlinear) it can be imagined that to solve optimization task is not an easy task. But if the nonlinear effects can be neglected, in principle, for individual robot arms, the well-known optimal “bang-bang” control principles could be applied. As far as we know, it is not very frequently applied in robotics. The reason is: **a robot is not an artillery gun, or a spacecraft, or any similar**.

In the present paper we will not follow the way outlined in this Introduction. In the second part of the paper the motion planning problems will be specified and analyzed. Also a state-of-the-art summarization is given. The 3rd part outlines the basic results concerning time-optimal cruising motions. In this part the basics of parametric method of motion planning are given, too. In part 4, time-optimal PTP motion is analyzed and solution method is presented. In part 5, realization aspects are outlined. In part 6, the interrelation of path, trajectory planning, and trajectory tracking is analyzed. In part 7, the problems of a very important practical field, the making of corsets for spinal diseases corrections is discussed. In Appendix, a simple example for time-optimal cruising trajectory planning is given.

ROBOT MOTION PLANNING

Now, let us return to the rather exact formulation of the robot motion planning problems. The following tasks should be solved:

- Path planning
- Trajectory planning
- Trajectory tracking

PATH PLANNING

Given a robot and its environment. The task is to plan a path which results in a transition of the end-effector centre point:

a.) from one position to another position;
b.) through a series of positions;
c.) along a continuous path.

During these actions it may also be required that the orientation of the grippers, or working tools attached to the end-effector have the given orientations. Sometimes, the path planning can be approached as a pure geometric problem, but in many cases, the path, trajectory planning and tracking problem are deeply interconnected. In the cases when these levels can be considered separately, for path planning, optimization problems, with geometric criteria, can be formulated. For example, the goal may be to get the shortest path to walk over a series of points, or avoid obstacles, or avoid obstacles by volumetric bodies, etc. The powerful apparatus of computer geometry can be used to great extent to solve these problems.

TRAJECTORY PLANNING

Given a path to be followed by the working point (end-effector centre-point) of a robot, and the corresponding orientations of tools attached to it. The dynamic characteristics of robot joints are known including the constraints on torques, forces available at the actuators. The limit values of the joints speeds, the limit values of speeds in Cartesian coordinate system are also given. Possibly, the same is given for accelerations.

Complex knowledge is available about the technological process characteristics (requirements, forces, etc.).

The most general and practical requirement is to find the motion giving minimum time for performing the task.
Other goal may be to find the motion requiring minimum energy.

TRAJECTORY TRACKING

The task of the trajectory tracking, as it was mentioned above, is to plan the control action that guarantees the realization of the desired trajectories with the necessary accuracies.

2. MINIMUM-TIME TRAJECTORY PLANNING. STATE-OF-THE-ART.

In the Introduction the minimum time motions were reviewed. Bellow, more details are given.
The time optimal control problems can be classified into three categories:
Motion on constrained path between two endpoints.
Motion in free workspace between two endpoints.
Motion in a free workspace containing obstacles.
Concerning the robot motion in free workspace, a number of results are available. In Geering, Guzzella, Hepner, Onder (1986), [3] it has been shown that the time optimal controls of motion in free workspace, are regularly that of switching nature. The maximum torques (forces) are switched for accelerating and decelerating in an appropriate manner. A huge number of papers were dealing with different aspects of the above problem. An overview can be found in S. K. Singh (1991). In Singh's paper a general numerical method to the solution of similar optimization tasks was proposed, too. Discretization and the use of non-linear programming method form the essence of this approach.
In many of the application problems the motion is constrained to a given path. Examples include arc welding , milling, grinding, painting, deburring using robots.
Several researchers have addressed the problem of this constrained motion of robots. Recently, it has turned out that the parametric description of the robot motion is one of the most promising way of the investigation of constrained motion. The most detailed outline of this method can be found in K. G. Shin, N.D.McKay (1991)[9].
When using the parametric method, the differential equations characterising the motion of the joints of an n-degree of freedom robot can be transformed to a form where instead of n joint coordinates ($q_1, q_2, \ldots, q_n$) only the one path parameter ($\lambda(t)$) is present. The n non-linear, coupled (second order) differential equation of joints motion is transformed to a second order non-linear differential equation formulated for one parameter. Shin, McKay, and others, based on the parametric description, proposed an approach to the solution of the time-optimal control of constrained motion of robots. Shin and McKay also used the parametric description method to the determination of other than the time-optimal motion. An example is the solution of optimal control problem using minimum energy criterion.
When using the method of parametric description, usually, the parameter is the length ($\lambda(t)$) along the path. In the present paper this approach will be used to a high extent, with the goal of investigating cruising motion rather than investigating dynamics. J. Podurajev and J. Somlo (1993) [10] used a parameter the time derivative of which is proportional to the square root of the entire kinetic energy of the robot mechanism. Using this parameter, the equation of motion becomes extremely simple. This approach made possible to develop optimal robot control according to energy criterion in a straightforward manner.
Later, in this paper we introduce the time-optimal cruising trajectory planning problem in detail and solve it. Shortly, we speak about cruising motion when a robot end-effector performs some application tasks and during that moves with velocity slowly changing absolute value. Later, it is shown that a cruising motion is time-optimal when at least one of the joint velocity values is at its limit value.
In Figure 2.1 transient and cruising motions are shown together.

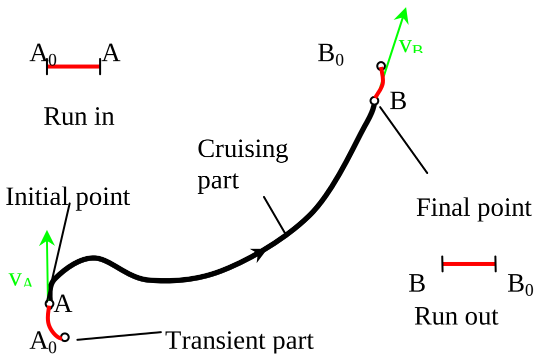

Figure 2.1
The cruising and transient parts of a path

In Somló J. and Poduraiev J. (1993) [11] a method is presented where, using the parametric method, it is solved that both acceleration and deceleration of the robot motion are at their limit values. in the transient motion parts (see also: Somló J., Lantos B.,P.T. Cat (1997) [2]).
The proposals for trajectory planning in technical literature are based on rather simple approaches. Below, the method proposed in K.S. Fu, R.C. Gonzalez, C.S.G. Lee (1987) [12] is discussed. (Similar approach is outlined in L. Sciavicco and B. Siciliano[13].)
At this approach, the coordinates of a series of points in Cartesian coordinate system. are given The corresponding joint coordinates values are determined by inverse

transformation. If the joint positions, speeds and possibly accelerations (deceleration) are known in the given points (and, also, the desired time of motion from point to point), the paths for joints satisfying the given conditions can be determined using proper-order splines.

For example, if in two points the $q_i(t_i), \dot{q}_i(t_i), q_i(t_{i+1}), \dot{q}_i(t_{i+1})$ joint coordinates and speed values are given, a third-order spline

$$q_i(t) = a_{0i} + a_{1i}t + a_{2i}t^2 + a_{3i}t^3 \tag{2.1}$$

may be used for path determination of the motion. Because

$$\dot{q}_i(t) = a_{1i} + 2a_{2i}t + 3a_{3i}t^2; \tag{2.2}$$

the $a_{0i}, a_{1i}, a_{2i}, a_{3i}$ parameter values can be determined from the 4 equations obtained at

$t=t_i$ and $t=t_{i+1}$.

When the accelerations are also specified, fifth-order spline with six adjustable parameters can be used. This method may also be used if N points, termed paths poits, are specified along the paths (see: [13].

These approaches are rather simple, but need some justification. Sometimes it may turn out that the trapezoidal speed profile is the adequate solution. This can be the case, for example, when the technological process constrains the speed. But, it can turn out only after the motion features were analyzed in detail. Indeed, in general, the speeds, the time of motion between the points, the acceleration, deceleration relations are unknown. In fact (see later) this quantities (among other factors) depend on the configuration of the paths. So, the proper order of investigations is to try to determine, in a systematic way, the above quantities, then go on with the solution of planning problems. The time-optimal cruising trajectory planning method outlined below solves these problems.

3. TIME-OPTIMAL CRUISING TRAJECTORY PLANNING

3.1 Motion on a given path

First we analyze the case when the path to move on is given. We assume that the acceleration, deceleration abilities of the robot are so high that the transient motion part (see: Figure 2.1.) may be neglected. This condition, usually, is valid for most of the industrial robots and applications. First, in the next paragraph of the present paper we will discuss one of the simplest cases to demonstrate the basic ideas.

3.1.1 Time-optimal cruising motion planning for polar manipulator.

In Figure 3.1, a 3-degrees of freedom cylindrical robot is shown. In Figure 3.2 the rotational and horizontal translation degrees are given. We name this mechanism as polar manipulator.

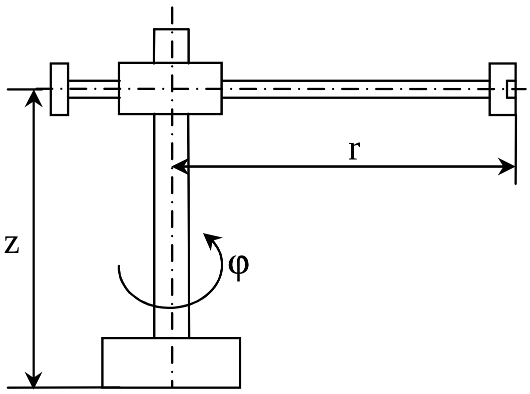

Figure 3.1
Cylindrical robot

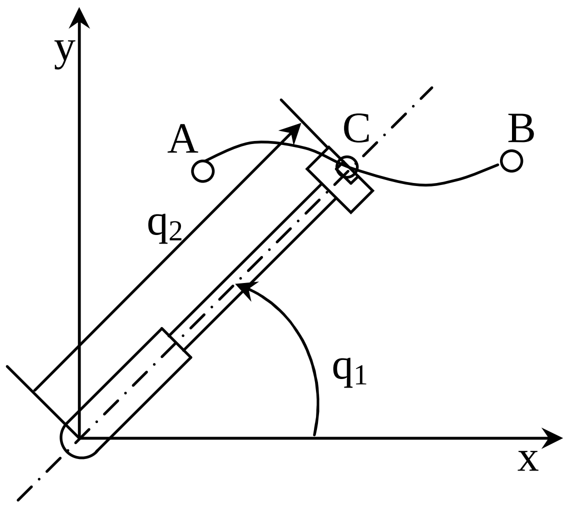

Figure 3.2
Polar manipulator

For time-optimal cruising trajectory planning we proposed a general method in [2 and 11]. We outline the basic idea of this method below. We want to move the end-effector working point (point C) from point A to point B along the path indicated in the Figure.

The equations of the direct geometry are:

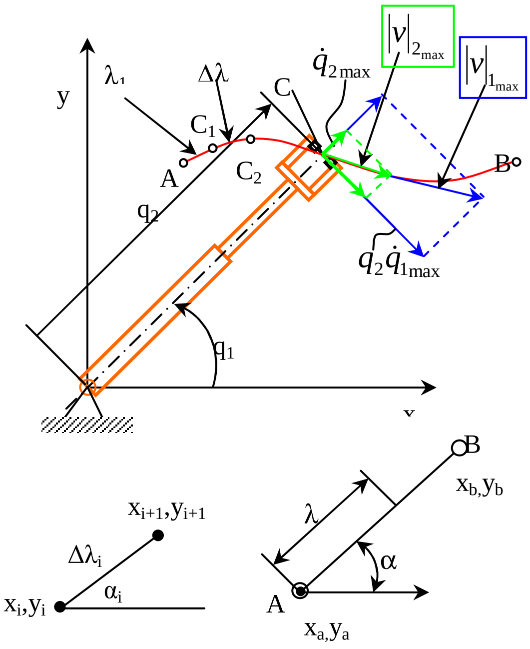

Figure 3.3
Time-optimal motion

$$x = q_2 \cdot \cos q_1$$
$$y = q_2 \cdot \sin q_1 \qquad (3.1)$$

To realize the motion we need the equations of the inverse geometry. These are:

$$q_1 = arc\tan\frac{y}{x}$$
$$q_2 = \sqrt{x^2 + y^2} \qquad (3.2)$$

In any given point, the velocity is directed tangentially to the path. The absolute value of the velocity is determined by the components given by the joints. Namely, the rotational joint results a component

$$|v|_1 = q_2\dot{q}_1 \qquad (3.3)$$

The translation joint motion results a component

$$|v|_2 = \dot{q}_2 \qquad (3.4)$$

The absolute value of the velocity is

$$|v| = \sqrt{|v|_1^2 + |v|_2^2} \qquad (3.5)$$

Now, let us try to determine the possible maximum absolute value of velocity. Let $\dot{q}_{1\max}$ and $\dot{q}_{2\max}$ be the maximum value of joint velocities. Clearly, in order to increase the absolute value of the velocity we should increase the joint velocities. Let us consider the case demonstrated in Figure 3.3.

Increasing $\dot{q}_2$ to his maximum value $\dot{q}_{2\max}$

$$|v|_{2\max} = \sqrt{(q_2\dot{q}_1)^2 + (\dot{q}_{2\max})^2} \qquad (3.6)$$

The components of the velocity vector are interconnected because the motion should be directed along the tangent to the path. So, to get $|v|_{2\max}$ is very easy as it is demonstrated on Figure 3.3.
(If analytical form is required, the following relation can be used to determine the required quantities

$$\frac{|v|_1}{|v|_2} = |\tan(\alpha_c - q_1)| \qquad (3.7)$$

Here α_c is the angle of the path related to axis x (see, α on Figure 3.3). It will be shown bellow that this step is unnecessary to perform.)
Increasing the absolute value of velocity more the maximum of other component may be reached

$$|v|_{1\max} = q_2 q_{1\max} \qquad (3.8)$$

It is clear that this value may not be realized because at that the other component would exceed its limit value.
So, the optimal velocity value is

$$|v|_{opt} = Min\left[|v|_{1\max}; |v|_{2\max}\right] \qquad (3.9)$$

In the given case this is $|v|_{2\max}$.
Let us now try to get an analytical expression for the realization of the above.
By the derivation of (3.1) we get

$$\dot{x} = -q_2[\sin(q_1)]\dot{q}_1 + [\cos(q_1)]\dot{q}_2$$
$$\dot{y} = q_2[\cos(q_1)]\dot{q}_1 + [\sin(q_1)]\dot{q}_2 \qquad (3.10)$$

Solving (3.10) for $\dot{q}_1$ and $\dot{q}_2$ we have

$$\dot{q}_1 = \frac{x\dot{x} + y\dot{y}}{\sqrt{x^2 + y^2}}$$
$$\dot{q}_2 = \frac{x\dot{y} - y\dot{x}}{x^2 + y^2} \qquad (3.11)$$

Let in a point (for example in C) world coordinates of which are x_i and y_i the angle of the tangent of the path with axis x be α_i (earlier we used for the same α_c). Then

$$\dot{x} = |v|\cos\alpha_i \text{ and } \dot{y} = |v|\sin\alpha_i \qquad (3.12)$$

Substituting the quantities into (3.11) we get

$$\dot{q}_1 = S_1(x_i, y_i, \alpha_i)|v|$$
$$\dot{q}_2 = S_2(x_i, y_i, \alpha_i)|v| \qquad (3.13)$$

Where S1 and S2 are the quantities obtained by the substitutions. So, for the maximum values we get

$$\dot{q}_{1\max} = S_1(...)|v|_{1\max}$$
$$\dot{q}_{2\max} = S_2(...)|v|_{2\max} \qquad (3.14)$$

Accordingly,

$$|v|_{1\max} = \frac{\dot{q}_{1\max}}{S_1(....)}$$
$$|v|_{2\max} = \frac{\dot{q}_{2\max}}{S_2(....)} \qquad (3.15)$$

For the determination of the optimal velocity value Equation (3.9) is valid.
According to the above, in every point of the paths the time-optimal cruising velocity can be determined. This velocity depends on the geometry of the paths (world coordinates of the points and direction of the tangents to the paths) and on the maximum possible values of the joint velocities.
We name ***dominant*** the joint the maximum velocity of which determines the optimum. The dominancy may change in some points. In these points both joints result the same maximum velocity (for the given example in these points $|v|_{1\max}$ and $|v|_{2\max}$ are equal).
In fact, Equations (3.10) and (3.11) are the rows of the Jacobian matrices. So, all the above can be interpreted from the point of view of using Jacobian matrices. This has been done in Somló, Lantos, P.T.Cat [2]. In this case we use

$$\dot{\mathbf{x}} = \mathbf{J}(\mathbf{x})\dot{\mathbf{q}}$$
$$\mathbf{x} = (x, y)^T$$
$$\dot{\mathbf{q}} = \mathbf{J}^{-1}(\mathbf{q})\dot{\mathbf{x}} \qquad (3.16)$$
$$\mathbf{q} = (q_1, q_2)^T$$

where **J** and $\mathbf{J}^{-1}$ are the Jacobian and inverse Jacobian matrices respectively.

3.1.2. Parametric method of trajectory planning

It was mentioned above that we propose to solve the time-optimal cruising trajectory planning for the whole path but how to do that was not outlined in a systematic way. In what follows, we will try to solve this task.
Shin and McKay in [7 and 8] proposed to use a parametric approach for robot planning problem. They proposed as a parameter the path length. It is clear that at any form of description of any path the world coordinates may easily be expressed as functions of path lengths. Indeed, the distance of any two points in plane or in 3D space may be easily computed. Representing the world coordinates of a robot as function of the determined paths lengths, the parametric description problem is solved. The inverse transformations connect the joint coordinates values with the world coordinates values. So, if the last ones are expressed by the parameter, it leads straight to the opportunity to express also the joint coordinates as functions of the parameter. This is the essence of the parametric approach. In fact, the parameter is an independent variable for the planning.
Let us return to the analyzed above example. Let the world coordinates in a point of the path be x_i and y_i Let in the next (nearest) point of the path the coordinates be x_{i+1} and y_{i+1} .. The distance between the two points is

$$\Delta\lambda_{i+1} = \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \qquad (3.17)$$

and

$$\alpha_i = \arctan\frac{y_{i+1} - y_i}{x_{i+1} - x_i} \qquad (3.18)$$

For the point with index (i-1) we have

$$\Delta\lambda_i = \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2} \qquad (3.19)$$

Let us assign to every point which we consider a serial number. This numbers are: i=1,2,…..N, and are also used as indices of the points. The last index value is N. The length of the path until point with index k is indicated as λ_k. Clearly λ_1 =0. For others

$$\lambda_k = \sum_{i=2}^{k} \Delta\lambda_i \quad (3.20)$$

In the last point of the path k=N and λ_N is the overall length of the path for which we will simply use λ.
In any point of the path the λ_k value can be computed. To λ_k belong the x_k and y_k world coordinates values. By inverse transformation the q_{1k} and q_{2k} values may be computed. In such a way the following functions may be determined:

$$\begin{aligned} q_1 &= f_1(\lambda) \\ q_2 &= f_2(\lambda) \end{aligned} \quad (3.21)$$

Of course, in this way we obtain functions determined in discrete points. This is very much suitable for robot control tasks where, usually, similar representations are used. But, if necessary, sometimes, analytical relations may also be developed. Let us consider the above example, and motion along a straight line (from A to B; Figure 3.3).
In this case

$$x = x_A + \lambda \cdot \cos(\alpha) \text{ and } y = y_A + \lambda \cdot \sin(\alpha)$$

$$q_1 = f_1(\lambda) = arc\ \tan \frac{y_A + \lambda\cos\alpha}{x_A + \lambda\sin\alpha} \quad (3.22)$$

and $q_2 = f_2(\lambda) =$

$$\sqrt{(x_A + \lambda\cos\alpha)^2 + (y_A + \lambda\sin\alpha)^2}$$

Assuming $\lambda = k \cdot \Delta\lambda$ (k=1,2,….N), we return to the discrete representation.
We remark that in this way (if necessary) sub-division of interpolation sections of paths is possible.
Now, we analyze the discrete velocity values in the selected points of the path.

$$\frac{\Delta q_1}{\Delta t_k} = \frac{\Delta f_1}{\Delta t_k} = \frac{f_1(\lambda_k + \Delta\lambda_k) - f_1(\lambda_k)}{\Delta\lambda_k}\frac{\Delta\lambda_k}{\Delta t_k}$$

and

$$\frac{\Delta q_2}{\Delta t_k} = \frac{\Delta f_2}{\Delta t_k} = \frac{f_2(\lambda_k + \Delta\lambda_k) - f_2(\lambda_k)}{\Delta\lambda_k}\frac{\Delta\lambda_k}{\Delta t_k} \quad (3.23)$$

It can be recognized that the velocity values may not exceed their limit values. So,

$$\dot{q}_{j\max} = \left(\frac{\Delta f_j}{\Delta t}\right)_{\max} \quad \text{(j=1,2)} \quad (3.24)$$

$$\frac{\Delta f_j}{\Delta\lambda} = \frac{f_j(\lambda + \Delta\lambda) - f_j(\lambda)}{\Delta\lambda} \quad \text{(j=1,2)} \quad (3.25)$$

$$|v|_{j\max} = \left(\frac{\Delta\lambda}{\Delta t}\right)_{\max} \quad \text{(j=1,2)} \quad (3.26)$$

Based on all above

$$|v|_{j\max} = \frac{\dot{q}_{j\max}}{\frac{\Delta f_j}{\Delta\lambda}} \quad \text{(j=1,2)} \quad (3.27)$$

Then,

$$|v|_{opt} = Min\{|v|_{j\max}\} \quad \text{(j=1,2)} \quad (3.28)$$

It is easy to recognize that if for a 3D problem we use the parametric representation, then we get

$$q_i = f_i(\lambda) \quad \text{(i=1,2,3)} \quad (3.29)$$

The derivatives of the joint coordinates may be obtained as

$$\frac{dq_i}{dt} = \frac{\partial f_i}{\partial\lambda}\frac{d\lambda}{dt} \quad \text{(i=1,2,3)} \quad (3.30)$$

$$\left(\frac{d\lambda}{dt}\right)_{i\max} = |v|_{i\max} = \frac{\dot{q}_{i\max}}{\frac{\partial f_i}{\partial\lambda}} \quad \text{(i=1,2,3)} \quad (3.31)$$

The $\frac{\partial f_j}{\partial\lambda}$ values may be computed as indicated above (see Equation (3.25)).
The time-optimal velocities are

$$|v|_{opt} = Min\{|v|_{i\max}\} \quad \text{(i=1,2,3)} \quad (3.32)$$

For the mathematical correctness of what mentioned above, it should be emphasized that the relations are meaningful in the points only where the derivatives used exist. So, the cross points of interpolation sections, the singularity points should be excluded from considerations. The above fact does not give any difficulty from the point of view of practical realizations.

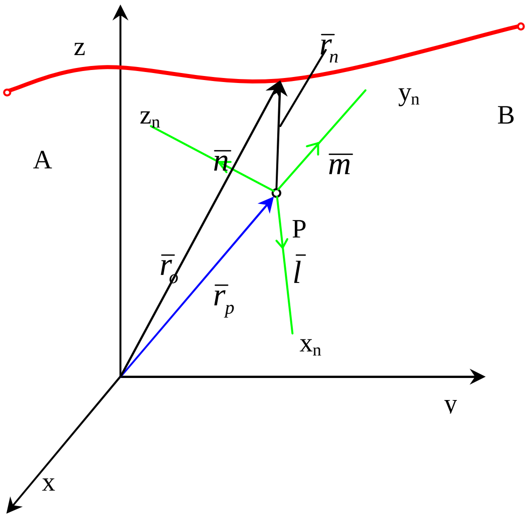

Figure 3.4
Changes of positions and orientations

Returning to the above, by determining the $|v| = \dot{\lambda}(\lambda)$ function for the whole path, in fact, we solve the trajectory planning problem. A graphic of a curve obtained for some planning tasks is demonstrated in Figure 3.5. This curve in itself is very much meaningful. At any λ value the velocity along the path may not be higher than the corresponding $\dot{\lambda}$ on the curve. On the contrary, any value below the curve may be applied. If a constant velocity along the path, is required for the whole path it may not be higher than $\dot{\lambda}_{min}$. In some cases, different constant velocities along the path sections may be required. These may be constructed using the curve. Other opportunities are possible, too.

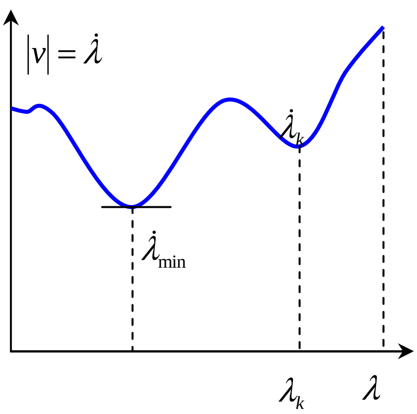

Figure 3.5
A $|v| = \dot{\lambda}(\lambda)$ function

But the most important fact is that using the planning result we can establish the motion time path length relation. Indeed,

$$\frac{d\lambda}{dt} = |v(\lambda)| \tag{3.33}$$

So

$$t(\lambda) = \int_0^{\lambda} \frac{d\lambda}{|v(\lambda)|} \tag{3.34}$$

By the use of the Equation (3.34) the time values belonging to any λ can determined and so the

$$q_1 = g_1(t) \text{ and the } q_2 = g_2(t) \tag{3.35}$$

input signals for the drives may be determined which realize time-optimal cruising motion.
An example for the polar manipulator is given in the Appendix.

3.1.3. The changes of position and orientation together

In the previous paragraph only the translation motion of the end-effector centre point was considered. It is well known, that together with the translation motions, depending on the path, on the robot construction and parameters, the orientation also changes. Requirements are formulated, very frequently, not only for translation but for orientation motion, too. The proposed method can be used for this case without any change.
The synchronised translation and orientation changes can be interpreted as shown in Fig. 3.6.

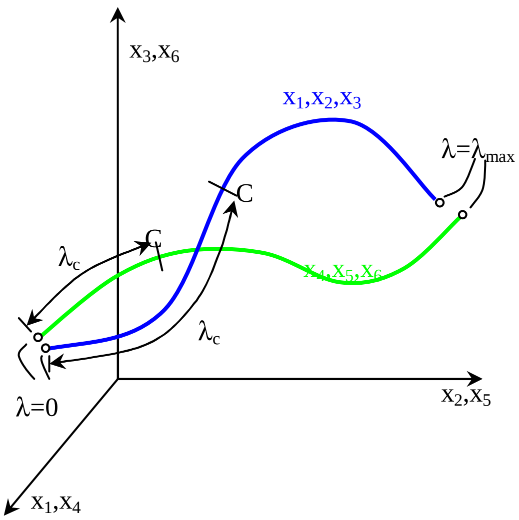

Figure 3.6.
Translation and orientation paths

The tool-centre point motion is characterized by vector $\mathbf{r_p}$. The tool-frame x_n, y_n, z_n is determined by its orthogonal unit vectors **l**, **m**, **n**. As it is well known there are only 3 independent values among the components of these unit vectors. So, if

$$\mathbf{l}=(l_x, l_y, l_z)^T \quad \mathbf{m}=(m_x, m_y, m_z)^T \quad \mathbf{n}=(n_x, n_y, n_z)^T \qquad (3.36)$$

Then, selecting any 3 independent components we can introduce the x_4, x_5, x_6 orientation (world) coordinates. To be consequent, we introduce also $x=x_1$, $y=x_2$, $z=x_3$. Solving the path planning problem, which now can be interpreted in spaces x_1, x_2, x_3 and x_4, x_5, x_6, we can solve the trajectory planning problem. For that we use the parametric method.
The parametric method of planning gives:

$$q_4 = f_4(\lambda),\ q_5 = f_5(\lambda),\ \text{and}\ q_6 = f_6(\lambda) \qquad (3.37)$$

We remark that, usually, in the equations of the inverse geometry, for q_4, q_5, q_6 the values of q_1, q_2, q_3 are involved. It does not give any difficulty when using the parametric method.
All the above is a clear indication for the fact that the time-optimal trajectory planning method and equations stay valid. That is Equations (3.31) and (3.32) results time-optimal motion with the only change

$$i= 1, 2, 3, 4, 5, 6 \qquad (3.38)$$

We remark that when 5D application problems are solved by the robots the orientation space becomes a plane. When the orientation should be kept constant the orientation curve becomes a point. Irrespective of the above, the proposed planning method stays valid.

4. TIME-OPTIMAL MOTION FROM POINT TO POINT

The point-to-point (PTP) motion is simpler to analyze and realize than that for motion along a path. Nevertheless, it is very important to understand and use time-optimal motions. When PTP motion is used, proper step functions are input to the actuators of the joints, and the realized motions of the joints are step responses. The motions in Cartesian coordinates systems are determined by the equations of direct geometry of the robot. If the transient processes are short and the joints motions realize maximum velocities, the motion is close to same time-optimal cruising one but with undetermined before geometry. Now, let us analyze the question what are the opportunities of realizing the point-to-point motion when the motion on different paths may be realized, too.

As an example, consider again the rotating and translating joint of a cylindrical robot (Fig. 3.2). Let us suppose that the transient motion is very fast. So, the motion features are determined basically by the cruising.
Let the task be to get from point A to B of Fig. 3.2. Then the difference of coordinates is:

$$\Delta q_1 = q_{1B} - q_{1A}; and\ \Delta q_2 = q_{2B} - q_{2A} \qquad (4.1)$$

Let the velocity limits be

$$\dot{q}_{1\max}\ and\ \dot{q}_{2\max} \qquad (4.2)$$

They are constant values, independent of the states. Let

$$t_{1\min} = \frac{\Delta q_1}{\dot{q}_{1\max}}\ and\ t_{2\min} = \frac{\Delta q_2}{\dot{q}_{2\max}} \qquad (4.3)$$

It is clear that only the bigger of the two values in (4.3) may be realized. This will give the absolute minimum of the motion time.
In formal terms

$$t_{\min} = Max(t_{i\min}) \qquad i=1,2 \qquad (4.4)$$

The joint which determines this minimum time is named *dominant.*
At minimum time motion the dominant joint will be at its maximum possible velocity in every moment.
The other joint's velocity value may vary in the domain

$$\dot{q}_{j\min} \le \dot{q}_i \le \dot{q}_{j\max} \qquad j=1,2 \qquad (4.5)$$

Now, let the dominant joint index be i and the non-dominant joint index j. Let us move the dominant joint with the velocity $\dot{q}_{i\max}$. It is clear that any velocity for the other joint in the $\dot{q}_{j\min}, \dot{q}_{j\max}$ domain (see (4.5)) may be applied. If moving from point A with $\dot{q}_{j\min}$ and $\dot{q}_{j\max}$ the borders of minimum time paths in the x,y plane may be determined. Using the equations of kinematics it is very easy to get the boundary curves. It is equally easy to get the boundaries for the motion from point B using the method of backward time. (see Fig. 4.5). If there exists a common, contiguous area inside the borders containing A and B, then every realizable trajectory of this area is a minimum time one. Any trajectory of the domain is

realizable if $q_i = \dot{q}_{i\max}$ and the (4.5) constraint (j=2) is satisfied.
Let us determine the border curves for the above example:
a.) Case when the rotation motion is dominant.
That is: $t_{1\min} > t_{2\min}$.
The speed of rotating joint should always be at its limit value.

$\dot{q}_1 = \dot{q}_{1\max}$.

For the motion of the translating joint one has the following limit values:
For the motion from point A

$$q_{2A} - \frac{q_1 - q_{1A}}{\dot{q}_{1\max}}\left|q_{2\min}\right| \le q_2 \le q_{2A} + \frac{q_1 - q_{1A}}{\dot{q}_{1\max}}\dot{q}_{2\max} \qquad (4.6)$$

For the motion from (to) point B

$$q_{2B} - \frac{q_{1B} - q_1}{\dot{q}_{1\max}}\dot{q}_{2\max} \le q_2 \le q_{2B} + \frac{q_{1B} - q_1}{\dot{q}_{1\max}}\left|\dot{q}_{2\min}\right| \qquad (4.7)$$

b.) Case when the translation motion is dominant.
That is: $t_{2\min} > t_{1\min}$.
The velocity of the translation motion should always be at its limit value.

$\dot{q}_2 = \dot{q}_{2\max}$

For the motion of the rotating joint one has the following limit values:
For the motion from point A:

$$q_{1A} - \frac{q_2 - q_{2A}}{\dot{q}_{2\max}}\left|q_{1\min}\right| \le q_1 \le q_{1A} + \frac{q_2 - q_{2A}}{\dot{q}_{2\max}}\dot{q}_{1\max} \qquad (4.8)$$

For the motion from (to) point B:

$$q_{1B} - \frac{q_{2B} - q_2}{\dot{q}_{2\max}}\dot{q}_{1\max} \le q_1 \le q_{1B} + \frac{q_{2B} - q_2}{\dot{q}_{2\max}}\left|\dot{q}_{1\min}\right| \qquad (4.9)$$

The realizable trajectories lie inside the border curves demonstrated on Fig. 4.1 and 4.2. All the trajectories for which in every point the following relation is fulfilled:

$$\dot{q}_{i\min} \le \dot{q}_i \le \dot{q}_{i\max} \qquad i=1,2, \qquad (4.10)$$

and have the given initial and final points are realizable trajectories.

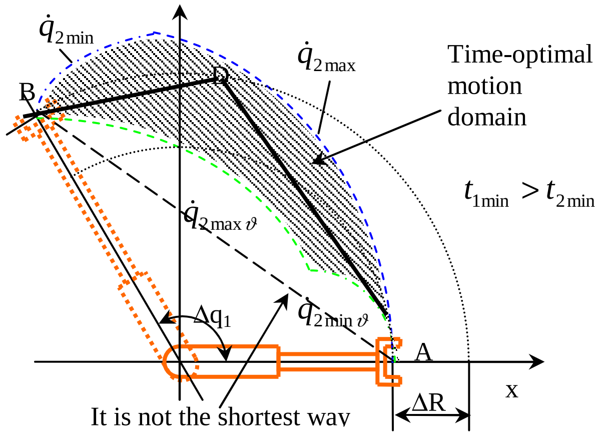

Figure 4.1
Time-optimal PTP motion

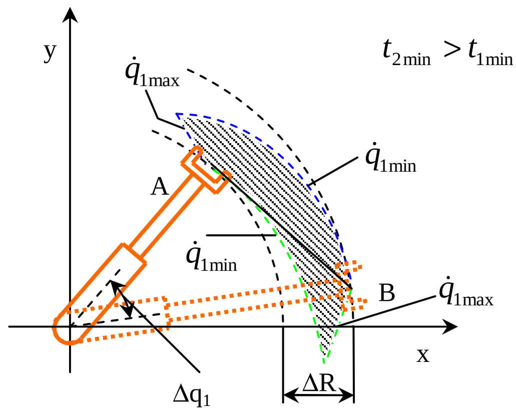

Figure 4.2
Time-optimal PTP motion

The border curves are also realizable trajectories. There is always a special realizable trajectory which results the termination of the motions for the different joints at the same time. In the case of the given example, it simply means that the quicker joint velocity should slow down to have $t_2 = t_{1\min}$ (or $t_1 = t_{2\min}$). This implies the corresponding trajectory inside the border curves.
It is an interesting question how to choose from the set of realizable trajectories. All these trajectories realize the minimum of the time of the motion. It is possible to find trajectories which have some features more favorable than the others. For example: it can be a criterion to find the minimum time trajectory realizable by the minimum of energy.
It is also an interesting question what is the geometry of the minimum time trajectories for different robots and different tasks. It can be seen in Fig. 4.6 that, for example, in the case of Fig. 4.6 b there is a straight line path which can be a minimum time, realizable trajectory. In the case of Fig. 4.6 a, such a line does not exist. The behavior of the robots from this point of view, including the investigation of the

time of motions on different arcs, was analyzed by H. Doghiem (1993 a, 1993 b) [14,15].

4.1. Case when the shortest way for a robot between two points is not the straight line

Looking at Fig.4.1 it can be recognized that if moving from A to B along the straight line it will take more time than moving on any time-optimal cruising trajectory. So, if we regard as the shortest way the one which takes the least time, it will not be the straight line. To handle the case is very easy. First we define the minimum time using Relation (4.4). Then we determine the time-optimal cruising trajectory planning for the straight line, which gives also the motion time, we get a clear picture about the quantitative characteristics.

4.2. Optimal PTP motion in space

What outlined for the example can be easily generalized for any degree of freedom. The dominant joint determination is exactly the same as in 2D case. Of course, when ND (N=3, 4, 5, 6) problems are considered in Equations (4.1) $\div$(4.5), the quantities with the proper indexation should be used. Furthermore, the individual representation of quantities for translation motion of the tool-centre point and orientation motions is necessary (as it was introduced for the investigation of time-optimal motions on a given path). Assuming that the dominant joint moves with the limit speed, the other joints have some freedom of motion the limits of which can be determined similarly as in the 2D example above. So, we move the dominant joint by the maximum velocity. Moving the other joints with $\pm$ limit velocities (from point A and with backward time from point B) we get sub-spaces which form time optimal domains. Any realizable trajectory of these sub-spaces is a time optimal trajectory. The optimal motion paths domains now become 3D sub-spaces for translation and orientation motions. It is not so easy as in the 2D case to analyze the processes but it is fully possible. Even, graphical representations may help to find the most suitable paths.

4.3. Optimal motions in free space with obstacles

An excellent opportunity to solve the motion planning problem in the presence of obstacles is given by the approach outlined above. In this case, it is highly proposed to use graphical representations of the time-optimal motion domains. If realizable paths exist in between the optimal border and the obstacles borders, all these are time-optimal, that is give the same minimum time. The final choice may be made by considering a number of other than the time aspects.

5. REALIZATION OF TIME-OPTIMAL MOTIONS

Let us deal at first with the motions in a given paths. As it is clear from the outlined the determination of the time-optimal trajectories is extremely simple. The equations which are necessary, for most of the industrial robots are known or can be easily derived. The maximums of joint velocities are included in the user handbooks of the robots. If in doubt, these values can easily be measured.

The optimal trajectory planning computations may be easily realized off-line (see: Appendix). Then, using the data, final choice of the velocities is possible with the guarantee of part processing times. These may prove much more favorable than the ones chosen without the knowledge of consequences of the planning decisions. For the realization of what above detailed user oriented, interactive computer programs can and were developed.

For the demonstration, let us consider the Figure in Appendix. It can be recognized that moving from point A until close to point C the velocity value along the path is close to $|v| = 0.1[m/\sec]$. In the region close to point D this velocity may reach a triple value. Many similar facts can be recognized at the solution of individual tasks. Conclusions may be deduced for application planning, concerning even the path planning issues,.

Returning to the possibility of realizing time-optimal (or close to that) motions, it can be stated that the modern industrial robot motion characteristics (velocities) are so high that even the minimum of time-optimal velocities is so high for the given paths that this constant velocity may be applied very effectively. But it is useful to know what it is.

One may never know when the big problems occur. So, if a robot control is equipped with an option which realizes time-optimal motion, it is a significant step forward. To demonstrate the opportunity of realizing such a control system, an open system architecture robot control device was developed and experimented by Sokolov, Somlo, Lukanyin as reported in publications [16 and 17].

It is remarkable that the time-optimal motion planning gives upper bounds for the velocities. The final choice of the velocity should take into account other constraints, too (see: Somlo, Lantos, P.T.Cat [2]). The other constraints may be grouped into: technological constraints, safety constraints, psychological constraints, etc.

Let us return to the question of free paths. For simplicity let us consider the 2D case, but suppose, that the device is able to realize not only PTP motions. Let us try to determine the path of the time-optimal motion. First (trivial) choice may be the straight line between A and B. If it is not time-optimal (see: Figure 4.1), we can try to choose two straight line sections (ADB in Figure 4.1). If these are realizable, they might construct the path. Application of third and higher order splines for motion planning was reviewed in paragraph 2.1. Splines may also be candidates for realizing time-optimal trajectories. For satisfying the time-optimality condition they should fully lei inside the optimal border, and the trajectories should be realizable.

6. PATH, TRAJECTORY PLANNING AND TRAJECTORY TRACKING TOGETHER

Path and trajectory planning tasks are strongly interconnected. If the planner allocates in the workspace of the robot some paths, it is a very easy task to determine the time-optimal cruising regimes. It is also possible to determine the corresponding $\dot{\lambda}(\lambda)$ functions. Estimating these functions gives a through picture about velocity characteristics. Velocities may be properly decreased, manipulated. Paths may be relocated in the workspace. Paths may be modified (for example, smoothed to get smaller velocity differences), etc.

Trajectory tracking should provide the realization of the inputs given by trajectory planning. We divide the problem into two parts. The first part is the transient part realization, while the second comprises the realization of the motion on the time-optimal cruising part. As it was mentioned, to analyze the motion on the transient part and find suitable (optimal) control laws are not easy tasks and hard to realize. Nevertheless, this field is full of nice results.

Below, we will deal only with the cruising part. As, for every point of the paths we have the joint coordinates, and correspondingly, all the derivatives of them, we can determine, (using the Lagrange equation (1.1), (1.2) (or others)) the torques (forces) necessary for the given motion. If all the torques (forces) are in the available regions (below the limit values) there is a good chance that the robot control will provide processes close to the required. This is especially true if sophisticated robot control methods are used (see: Asada, Slotine [1], Somlo, Lantos, P.T. Cat [2], Fu, Gonzalez, Lee [12], Sciavicco, Siciliano [13] and many others).

Concerning trajectory tracking, the control science provides effective methods for the solutions. For example: Computed Torque, Model Reference Adaptive , Sliding Mode Control may be used (see, for example: [1,2,4,5,12,13].

7. ONE FIELD OF POSSIBLE APPLICATIONS

To use low depth very high feed motions is a great promise for many technological processes. We are concerned with the problem of making corsets for spinal disorders (see: Somló, Tamás, Halász [20] and others, for example, [18] and [19]) The aim of the works is to develop an "intelligent corset" (plastic girdle) and a medical examination appropriate for early recognition and treatment of spinal disorders of young people aged 8-18 including a multidisciplinary methodology on the basis of medical science, informatics and material technologies. These research tasks also include the investigations of the opportunities of using advanced CAD/CAM technologies for the realization of the corsets. Corsets are made from plastic sheets that can not be produced by classical CNC machine tools technologies. Usually, dies are produced which serve for making the parts by pressing or by vacuum forming, etc. Recently, a new technology called. DSF (Dieless Sheet Forming; sometimes ISF (Incremental Sheet Forming) is used). has appeared to manufacture individual free-form sheets (In the literature Sheetmetal forming is in use but we intensively left out the metal attribute.) The principle was published in the patent of E. Leszak in 1967 [21]. EU Sixth Framework Project (Contract:014026) was devoted to the topic [22]. A survey of Japans results is published in [23].

In Figure 7.1 DSF process, equipments and product is demonstrated.

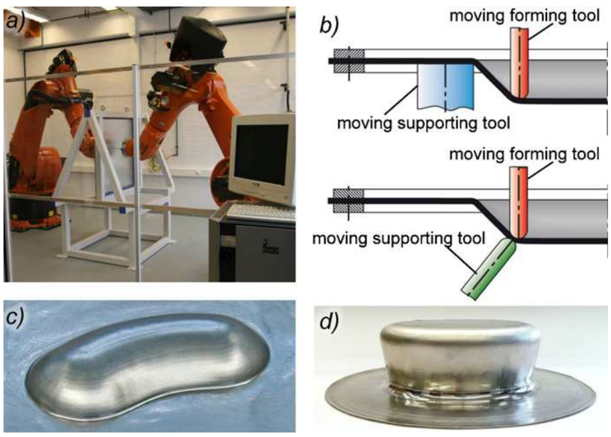

Figure 7.1
DSF process and equipment

In Figure 7.2 one-sided forming with warming is shown.

The DSF technology is very similar to CNC manufacturing processes. It is similar, most typically, to vertical milling of sculptured surfaces by finger-like milling tool. However, no cutting but pressing or hitting of the surface is performed.

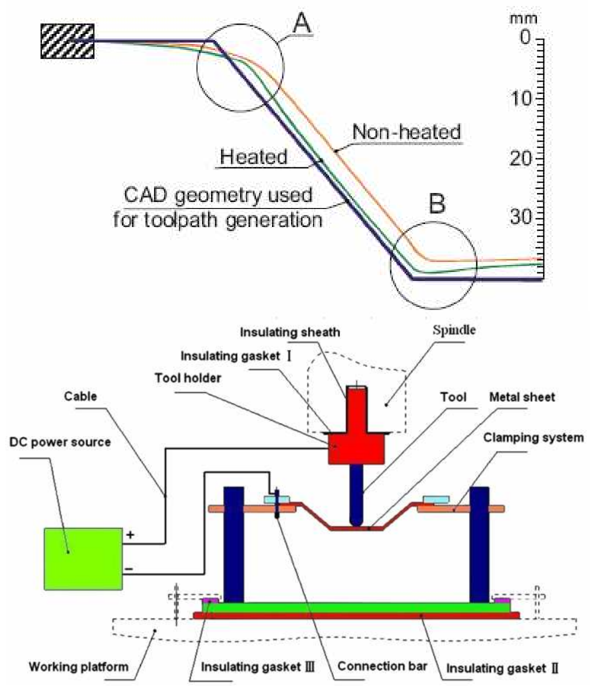

Figure 7.2
One-sided forming with warming

In our research work polyethylene sheets were formed. The reason was, as it was noted above, that such sheets might be useful at corset making.
In Figure 7.3 a corset and a girl with spinal disorder are shown.

Figure 7.3
A corset and a girl with spinal disorder

Prior to our experiments, we could get information on DSF of polymer materials in cool condition only. For us it was clear, mostly from the practice of corsets making, that in our case warming was necessary. It was a long way to find the proper temperatures and processing data. After finding the proper regimes, we could realize simple parts (truncated cones) in high quality. Then, we began to produce trough-like polyethylene parts. The thickness of sheets was $3 \div 5$ mm. The experience gained by the cones was very useful. In a very short time we could find the proper warming regime and data to get good-quality parts. In this case we faced other problems. The method of fixation of the part affected the quality very much. The sheet should have fixation stripe close to the forming actions. This is an indication for the fact that if no two- sided processing is used, some additional measures are necessary to provide for satisfying the requirements. After solving all these problems the trough-like part making provided suitable quality. But the time of production was not short enough. For wide practical use the production time should be significantly decreased.
It seems to us that the given technological process is very much suitable for trying to use very high, possibly time-optimal velocities.
One of the reasons for that is that the contact forces are extremely low.
Similar conditions are valid when sculptured parts are produced from polyfoam materials by cutting. This is also used at corset making. Polyfoam torsos are manufactured by CNC machine tools which serve for vacuum forming of corsets. These torsos may also be produced by high velocity cutting realized by robots.

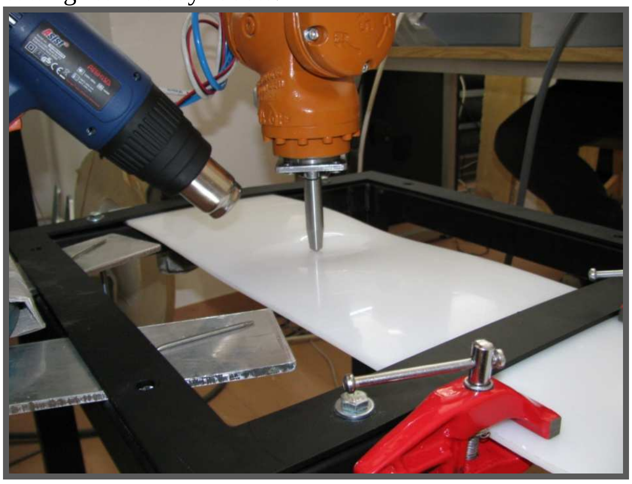

Figure 7.4
DSF of polyetilen sheets by warming

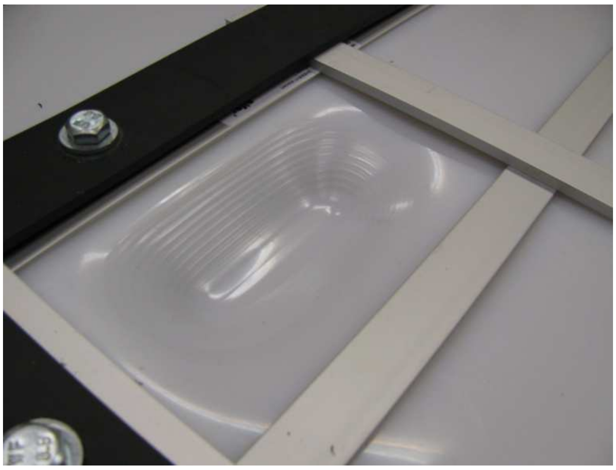

Figure 7.5
Through-like part made by DSF

8 CONCLUSIONS

Proposals in the paper give a general and simple to- realize method for time-optimal cruising trajectory planning for industrial robots. The proposed approach is based on the parametric method. All the parameters which are needed for the application (for example, joint velocities limit values) are easily available. The basic relations reflecting the essence of the approach are given by Equations (3.29) ÷(3.32). Then, determining the $\dot{\lambda}(\lambda)$ function and from that the $t = t(\lambda)$ relations, the joint drives inputs may readily be determined and consequently the time-optimal motion may be realized. A slightly different but in the spirit close method can be used for free paths.

The time-optimal trajectory planning method provides plenty of information for application planning. Existing applications' time needs may be shortened, and new applications may be developed with outstanding characteristics.

8.1. Some research topics

It is reasonable to reconsider existing robot applications and estimate the opportunity of decreasing times.

It is reasonable to relocate the paths in workplace (for existing robot applications) to find out their effect on production time (where to put?).

It is an interesting research problem how close the optimal processes may be approximated by robots having rather simple control opportunities. For example, if piece-wise constant velocities are used for shorter or longer sections (how to program?).

Many other similar problems may be formulated all giving practical advance with almost no price at all.

APPENDIX

Let us consider a polar manipulator which is given in Figure A1. The shape of the path to be realized is also given (the proportions are not real). The circle centre point data are not indicated because the circle is tangent in point B to line AB.

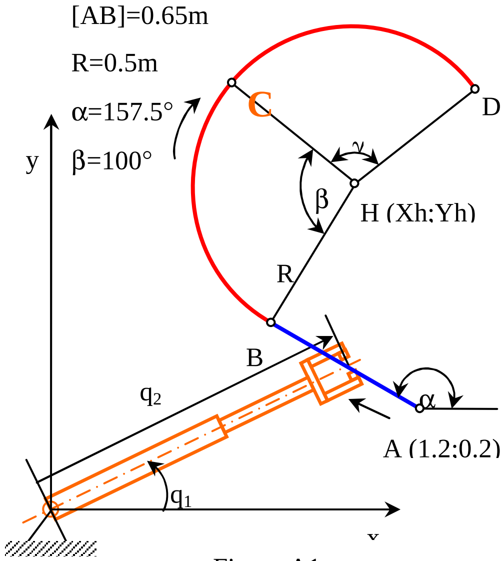

Figure A1
Polar manipulator and given path

We determined the time-optimal cruising trajectory according to the proposed in the paper. The following observations can be concluded. The motion begins at point A with the maximum (negative) speed for the translation joint. Close to point B (still on the linear part of the path) the maximum speed "switches" to the rotation joint. Then, the optimal motion velocity is determined by $\dot{q}_{1\max}$. The length of this section is about 3 sec. The dominancy after about 7 sec of the overall motion switches again to the translation joint and a long motion section with $\dot{q}_{2\max}$ velocity is realized, until, close to point D, again the rotation joint becomes dominant.

It is easy to recognize that if we use a constant velocity along the whole path its value should be below v=0.1[$^{m}/_{sec}$]. Close to point D much higher velocities may be used. The overall time of the motion along the path at time-optimal motion is about 17 sec. If v=0.1[$^{m}/_{sec}$] is applied, this time is 27,4 sec. So, the economy is significant.

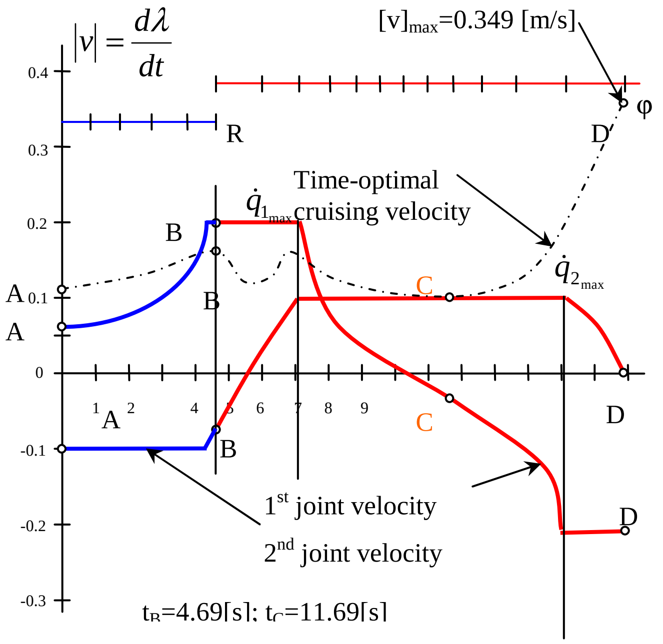

Figure A2
Time-optimal cruising trajectory

It is noteworthy that it was not mentioned here that the velocity limits of the joints should have constant values. For example, a real constraint might be given by the restriction of the centrifugal force acting on the load. These can be derived from the requirements of the safe gripping.
Let

$$q_2(\dot{q}_1)^2 < \frac{GF}{m_p} \qquad \text{(A.1)}$$

where GF- is the necessary gripping force,

m_p - loading mass.

At certain points of the path, (A.1) may give more severe velocity limit than the manual given constraint.

Acknowledgement

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